# B.Tech. Degree Examinations: November 2013

## Sub: Design of Prestressed Concrete (CE41T04)

## Class: B.Tech. Civil Engg. (VII Sem.)

### KEY

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### Section – 1

a) In the anchorage zone of a post tensioned PSC beam , what is the stress distribution?

**Ans: Triaxial**

b) What type of stresses induced by Circular prestressing prestressed concrete tanks induces

**Ans: Hoop compression**

c) The economical proportion of diameter to height of circular cylinder prestressed concrete tanks is

**Ans: 4:1**

d) If a simply supported concrete beam, prestressed with a force of 2500 kN is designed by load balancing concept for an effective span of 10 m and to carry a total load of 40 kN/m, the central dip of the cable profile should be

**Ans:** $2500 \times e = \frac{40 \times 10^2}{8} \Rightarrow e = 0.2m \Rightarrow$ central dip

$\therefore$ central dip of the cable $= \underline{\underline{200 \text{ mm}}}$.

e) A prestressed concrete beam of cross sectional area 'A', moment of inertia 'I', distance of extreme top fibre from NA is $y_t$, and that bottom fibre is $y_b$ is subjected to a prestressing force such that stress at top fibre is zero. What is the value of eccentricity ('r' is the radius of gyration)

**Ans:** $f_t = \frac{P}{A} - \frac{Pe}{I} \cdot y_t = 0 \Rightarrow \boxed{e = \frac{I}{A} \cdot \frac{1}{y_t} = \frac{r^2}{y_t}}$

f) List the losses existing only in post tensioned members

**Ans:** i) loss due to Elastic shortening if ~~small~~ the wires are successively tensioned
ii) Relaxation of stress in steel
iii) Shrinkage of concrete (v) Friction
iv) creep of concrete (vi) Anchorage slip.

g) A pre tensioned concrete member of section 200 mm x 250 mm contains tendons of area 500 mm$^2$ at centre of gravity of the section. The prestress in the tendons is 1000 N/mm$^2$. Assume modular ratio as 10, the stress in concrete is

Ans:- $f_c = \frac{P}{A} + \frac{Pe}{I} \Rightarrow \frac{(500 \times 1000)}{200 \times 250} = \underline{\underline{10 \text{ N/mm}^2}}$.

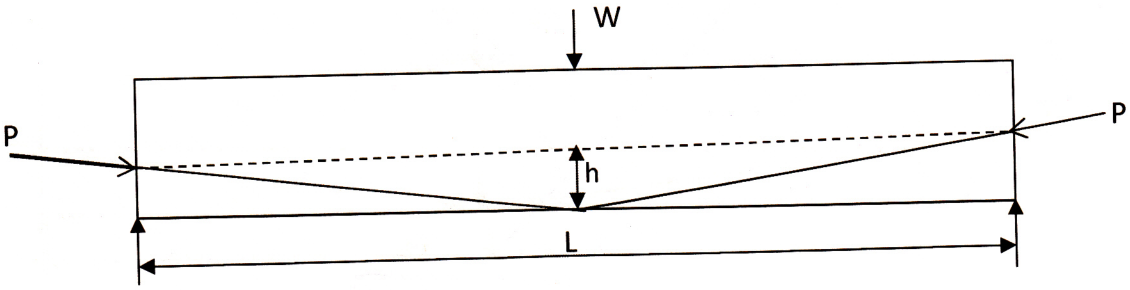
h) The partial safety factor for strength of concrete at limit state of collapse is

**Ans: 1.5**

i) The partial safety factor for combined dead, live and wind loads at the limit state of serviceability is

**Ans: 1.0, 0.8, 0.8**

j) The prestressing force in the tendon is P for the beam shown in the figure. The central dip 'h' required to fully balance a concentrated load W applied at mid span is

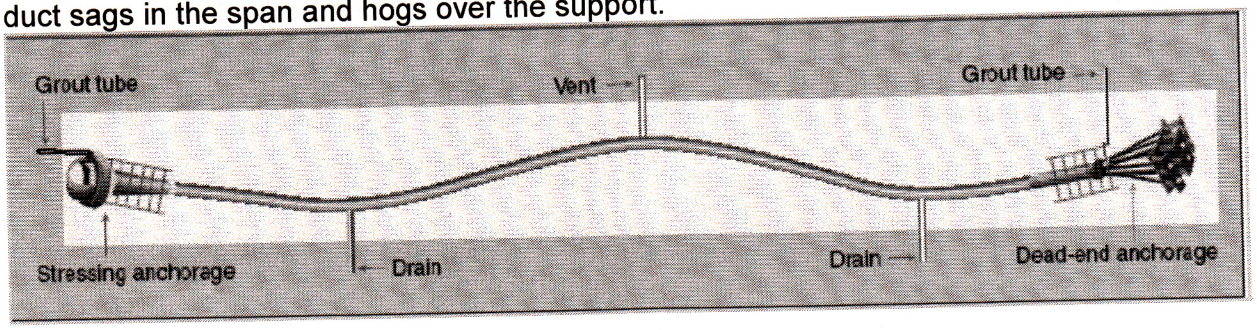



**Ans:** $h = \frac{WL}{4P}$

## Section - II

(2)(A) In post-tensioning systems, the ducts for the tendons (or strands) are placed along with the reinforcement before the casting of concrete. The tendons are placed in the ducts after the casting of concrete. The duct prevents contact between concrete and the tendons during the tensioning operation.

Unlike pre-tensioning, the tendons are pulled with the reaction acting against the hardened concrete. If the ducts are filled with **grout**, then it is known as bonded post-tensioning. The grout is a neat cement paste or a sand-cement mortar containing suitable admixture. The grouting operation is discussed later in the section. The properties of grout are discussed in Section 1.6, "Concrete (Part-II)". In unbonded post-tensioning, as the name suggests, the ducts are never grouted and the tendon is held in tension solely by the end anchorages. The following sketch shows a schematic representation of a grouted post-tensioned member. The profile of the duct depends on the support conditions. For a simply supported member, the duct has a sagging profile between the ends. For a continuous member, the duct sags in the span and hogs over the support.




The various stages of the post-tensioning operation are summarised as follows.

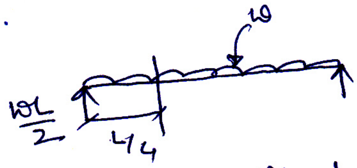
1) Casting of concrete.
2) Placement of the tendons.
3) Placement of the anchorage block and jack.
4) Applying tension to the tendons.
5) Seating of the wedges.
6) Cutting of the tendons.

Given data.

size : 200 × 300 mm   L = 6 m   $w_L$ LL = 4 kN/m   $\gamma_c$ = 24 kN/m³

self wt  $w_d = 0.2 \times 0.3 \times 24 = 1.44$ kN/m.

$$M = \frac{3(1.44+4)\times 6^2}{32} = \underline{18.36 \text{ kN-m}}$$




$$M = \frac{wL}{2}\times\frac{L}{4} - \frac{wL}{4}\times\frac{L}{8} = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{3wL^2}{32}$$

a) P (concentric) for zero fibre stress at the soffit

$$\frac{P}{A} - \frac{M}{Z_b} = 0 \Rightarrow P = \frac{M}{Z}\cdot A = \frac{18.36\times10^6}{3\times10^6}\times 6\times10^4 = 367.2\times10^3 \text{ N}$$

$$\boxed{P = 367.2 \text{ kN}}$$

$Z = \frac{1}{6}bd^2 = 3\times10^6$ mm³

$A = 200\times300 = 6\times10^4$ mm²

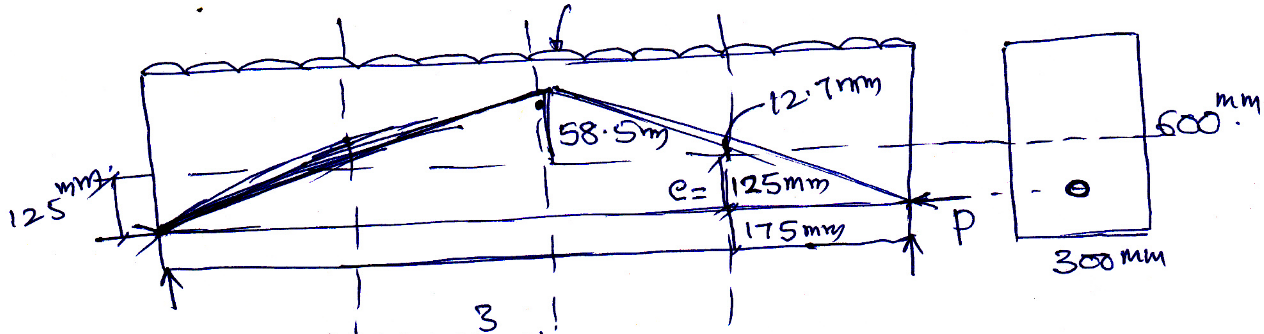
(b) e = ~~100~~ 50 mm

$$\frac{P}{A} + \frac{Pe}{Z_b} = \frac{M}{Z} \Rightarrow P\left[\frac{1}{6\times10^4} + \frac{50}{3\times10^6}\right] = \frac{18.36\times10^6}{3\times10^6}$$

$$\boxed{P = 183.6 \text{ kN}}$$

5) (b) × (D)

300 × 600 mm.   L = 12 m.




$P = 800 \times 2000 = 1600\times10^3$ N

$w_{LL} = 12$ kN/m   $w_D = 4.32$ kN/m   $w = 16.32$ kN/m

BM  At Mid span $M_c = \frac{16.32\times12^2}{8} = 293.76$ kN-m.

At quarter span $M_\phi = \frac{3}{32}\times wl^2 = 220.32$ kN-m.

At end  $M_e = 0$.

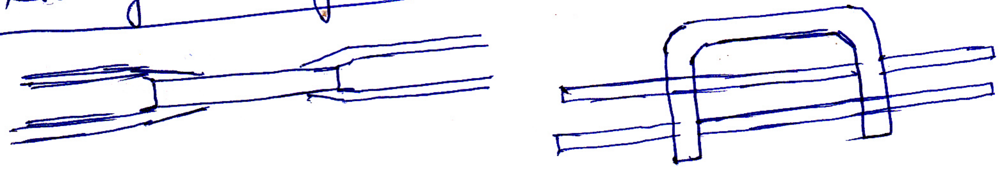
Location of the pressure line

At Mid span $e' = \frac{M}{P} - e = \frac{293.76}{1600} - 0.125 = 0.0586$ m $= 58.6$ mm

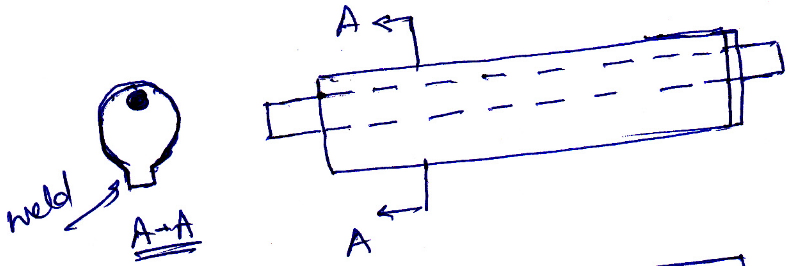
At Quarter span $e' = 0.0127$ m $= 12.7$ mm.

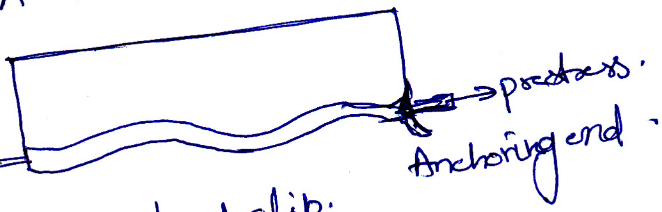
At ends  $e' = -e = -0.125$ m $= 125$ mm (below cg).

## Supplementary Anchoring devices



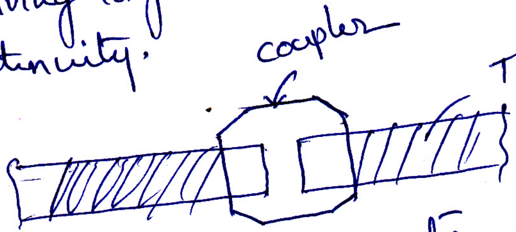
Weinberg Anchor slip



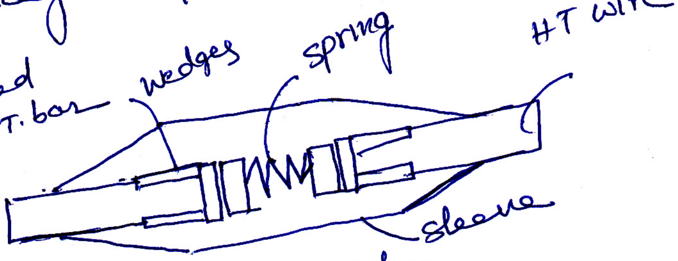




Dorland clip.

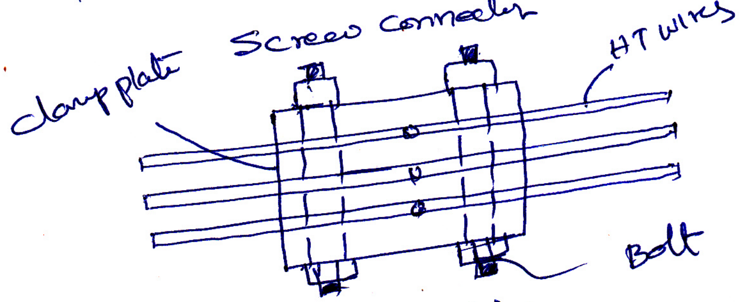
③ Tendon splices : In case of continuous prestressed Concrete members involving long tendons, it is necessary to splice the tendons to achieve continuity.



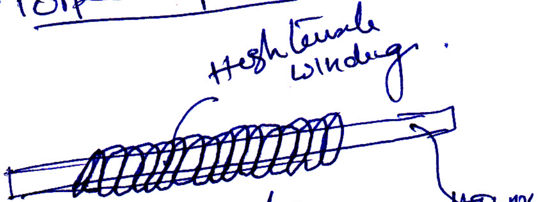

Screw connector




Torpedo splice




clamp splice




Wrapped splice

Screw connectors are normally employed to splice large diameter high tensile bars which can be threaded at the ends.

Torpedo-splice consists of tripple wedges for securing the wires and the entire unit is covered and protected by sleeve. This type of splice is largely used for splicing cold drawn wires, which are adopted for circular prestressing of tanks.

clamp splices are equipped with bolts and nuts, with a series of clamp plates to house the tendons b/w them.

## Types of Losses:

**pretensioning**

i) Elastic deformation of Concrete
ii) Relaxation of stress in steel
iii) shrinkage of Concrete
iv) Creep of Concrete

**post tensioning**

i) No loss due to elastic deformation if all the wires are simultaneously tensioned. If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete.
ii) Relaxation of stress in steel
iii) shrinkage of Concrete
iv) creep of Concrete
v) Friction
vi) Anchorage slip.

## Elastic deformation of Concrete

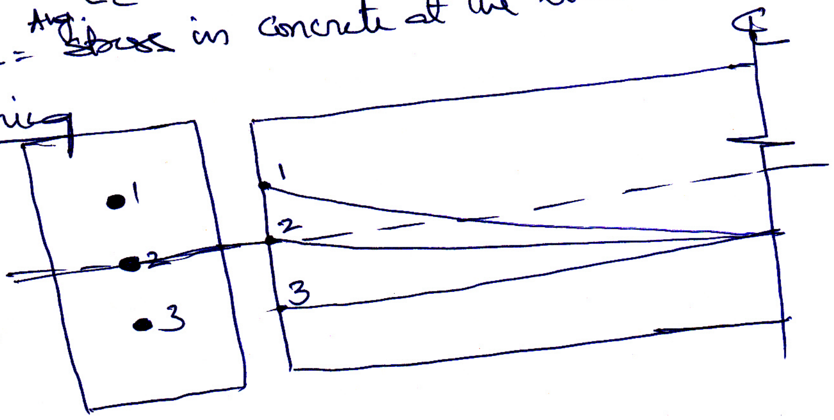
Depends on modular ratio and average stress in concrete at the level of steel.

loss of stress in steel $= \alpha_c \cdot f_c$.

where $\alpha_c = \frac{E_s}{E_c}$ : modular ratio

$f_c =$ Avg. stress in concrete at the level of steel.

## Due to Successive tensioning





$f_{c\,avg} = f_1 + \frac{2}{3}(f_2 - f_1)$.

loss of stress in steel $= \alpha_c \cdot f_{c\,Avg}$.

## Due to Shrinkage

Max shrinkage strain $\epsilon_s = 300\times10^{-6}$ m/m for pretensioning

$= \frac{200\times10^{-6}}{\log_{10}(t+2)}$ for post tensioning

t = Age of Concrete at transfer in days.

loss of stress = $\epsilon_{cs}\times E_s$.

where $E_s$ = Modulus of Elasticity of steel.

## Due to creep

i) ultimate creep strain Method.

ii) creep coefficient method.

$$\epsilon_c = \phi\times\frac{f_c}{E_c} \quad \text{where } \phi = \frac{\epsilon_c}{\epsilon_e} = \frac{\text{creep strain}}{\text{Elastic strain}}$$

Loss of stress in steel = $\phi f_e \alpha_c$.

## Due to Relaxation of stress in steel

It is provided as a % of initial stress in steel. The I.S. code recommends a value varying from 0 to 90 N/mm² for stresses in wires varying from 0.5 fpu to 0.8 fpu.

## Due to Friction

i) loss of stress due to the curvature effect — Tendons alignment which generally follows a curved profile along the length.

ii) Loss of stress due to the wobble effect — local deviations in the alignment of the cable.

The Magnitude of the prestressing force $P_x$ at a distance 'x' from the tensioning end is

$$P_x = P_0 \cdot e^{-(\mu\alpha + kx)}$$

where $P_0$ = Prestressing force at the Jacking end

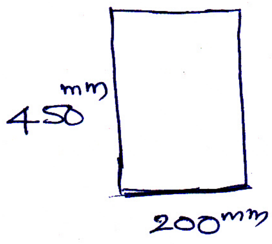
$\mu$ = coeff of friction b/w cable and duct.

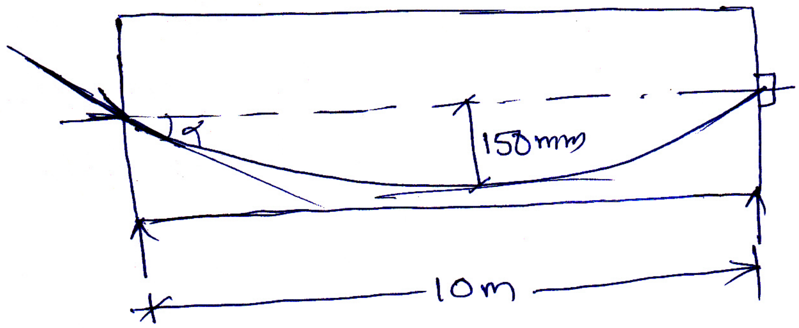
$\alpha$ = cumulative angle in rad through which the tangent to the cable profile has turned b/w any two points under consideration.

K = friction coefficient for wave effect.

e = 2.7183

$$\% \text{ loss} = \frac{P_0 - P_x}{P_0}\times100$$




$A_p = 800\,mm^2$

total wires = 24

Successive tensioning is 2 wires at a time.

$f_1 =$

$f_2 =$

Initial prestress $= 840\,N/mm^2$ (After anchoring):

~~Loss due to friction:~~

R is the Radius of the cable

Then $(R-0.15)^2 + 5^2 = R^2 \Rightarrow R = 84m$.

$\sin\alpha = \frac{5}{84} = 0.06$ rad.

Cumulative angle b/w tangents to the cable at supports $= 2\times 0.06$
$= 0.12$ rad.

Loss due to anchorage slip $= \frac{\Delta L}{L} \times E_s = \frac{1.25}{10\times 10^3} \times 210\times 10^3 = 26.25\,N/mm^2$

∴ Initial stress in the cable just ~~before~~ anchoring $f_2 = 840 + 26.25$

$f_2 = 866.25\,N/mm^2$.

∴ $f_1 = f_2 +$ frictional loss.

<u>Loss due to friction</u>

$P_x = P_0 (1 - e^{-(\mu\alpha + kx)})$

$= P_0 (1 - (\mu\alpha + kx))$

$= 866.25(1-(0.6\times 0.12 + 0.003\times 10)) = 777.89\,N/mm^2$.

∴ Loss due to friction $= 866.25 - 777.89 = 88.36\,N/mm^2$.

∴ $f_1 = 866.25 + 88.36 = \underline{954.61\,N/mm^2}$.

loss due to shrinkage $= 0.0002\times 210\times 10^3 = 42\,N/mm^2$

Loss due to Relaxation of steel $= \frac{3}{100}\times 866.25 = 25.98\,N/mm^2$

Total losses $= 88.36 + 26.25 + 42 + 25.98 = 182.59\,N/mm^2$

Final design stress $= 866.25 - 182.59 = \underline{683.66\,N/mm^2}$

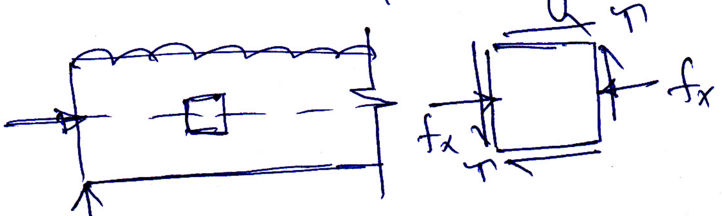
2(a) Three ways of improving shear resistance

(i) Horizontal (or) axial prestressing.

(ii) prestressing by inclined or sloping cables and

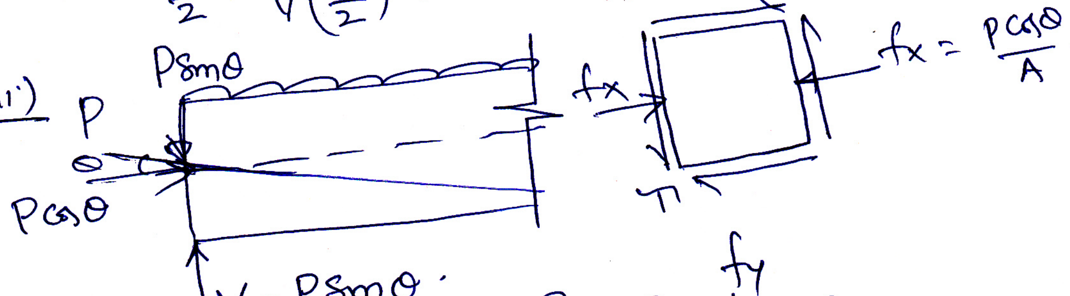
iii) vertical (or) transverse prestressing.

Case (i)



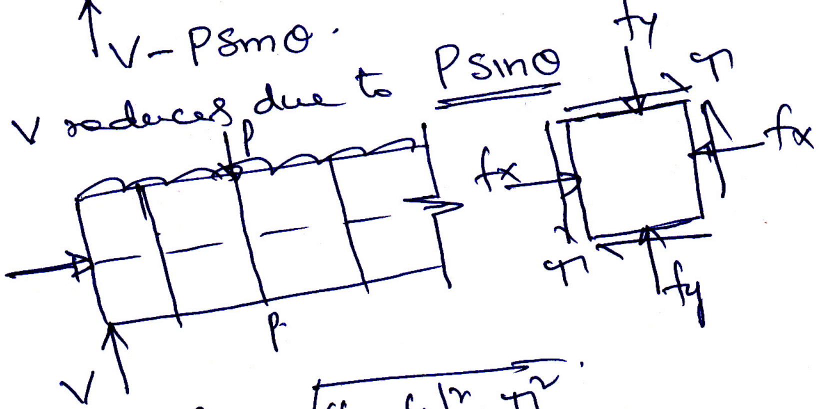

$$\sigma_2 = \frac{f_x}{2} - \sqrt{\left(\frac{f_x}{2}\right)^2 + \tau^2}.$$

Case (ii)



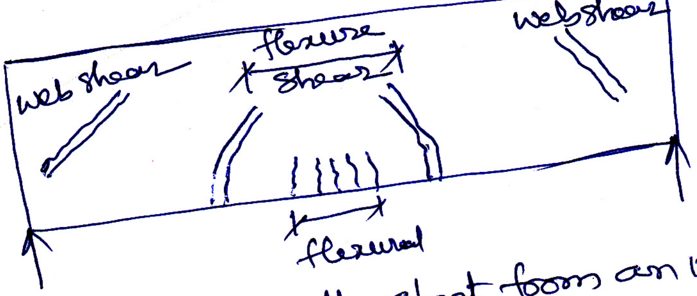

Net V reduces due to P sinθ

Case (iii)




$$\sigma_2 = \frac{f_x+f_y}{2} \pm \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau^2}$$

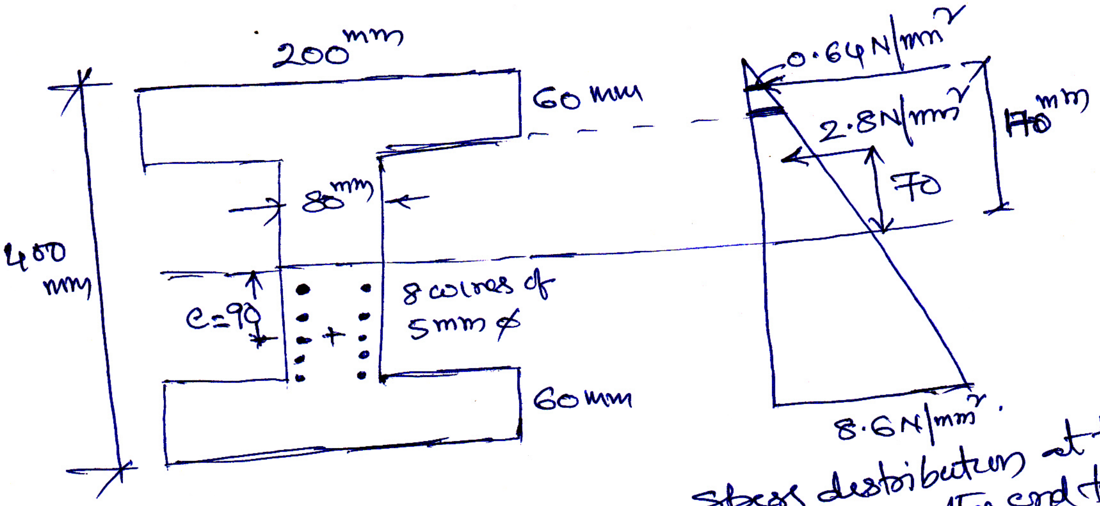
(b) Types of shear cracks




Web shear cracks generally start from an interior point, when the local principal tensile stress exceeds the tensile strength of concrete. Web shear cracks are likely to develop in highly prestressed beams with thin webs. particularly when the beam is subjected to large concentrated loads near a simple support.

Flexure shear cracks are first initiated by flexural cracks in the inclined direction. Flexure shear cracks develop when the combined shear and flexural tensile stresses produce a principal tensile stress exceeding the tensile strength of concrete. In members without shear reinforcement, the inclined shear cracks extend to the compression face resulting in sudden explosive failure. This is sometimes referred to as the diagonal tensile mode of failure.

(9)




Stress distribution at transmission length from the end face

$A_c = 46400\ mm^2$

$I = \frac{1}{12}\left[200 \times 400^3 - 120 \times 280^3\right] = 847.146 \times 10^6\ mm^4$

$Z_t = Z_b = Z = 4.235 \times 10^6\ mm^3$

Total $P = 8 \times \frac{\pi}{4} \times 5^2 \times 1280 = 201\ kN$.

stress at the bottom fibre $= \frac{P}{A} + \frac{Pe}{Z} = \frac{201 \times 10^3}{46400} + \frac{201 \times 10^3 \times 90 \times 200}{847.146 \times 10^6}$

$= 4.33 + 4.27 = 8.6\ N/mm^2$ (comp)

stress at top fibre $= 4.33 - 4.27 = 0.06\ N/mm^2 \simeq 0$

The Internal Moment due to the distribution of prestress developed is obtained as

$M = \left[200 \times 60 \times 0.64 \times 170 + 140 \times 80 \times 2.8 \times 70\right] = 351 \times 10^4\ N\text{-}mm$.

Transmission length $L_T = \sqrt{\frac{\sqrt{f_{cu}} \times 10^3}{0.0235}}$ for 5 mm diameter wires.

$= \sqrt{\frac{\sqrt{42} \times 10^3}{0.0235}} = 525\ mm$.

Max vertical tensile stress near the end face

$$= \frac{10M}{b_w h L_T} = \frac{10\times 351\times 10^4}{80\times 400\times 525} = 2.09\ N/mm^2.$$

Area of vertical reinforcement $A_{sv} = \frac{2.5M}{f_y h} = \frac{2.5\times 351\times 10^4}{140\times 400}$

$$= 158\ mm^2.$$

provide 3 bars of 6mm diameter stirrups (two legged) in the transfer zone.

(10) <u>Given data</u>

$D = 1000\ mm$ $\quad f_{ct} = 15\ N/mm^2$ $\quad W_w = 1.5\ N/mm^2$

$f_{min,w} = 2\ N/mm^2$ $\quad t = 75\ mm$ $\quad f_s = 1000\ N/mm^2$

$L = 6\ m.$

(a) circumferential wire winding

Comp stress in concrete $f_c = \frac{Nd}{\eta t} + \frac{f_{min,w}}{\eta}$

$$= \frac{(1.5\times 1000/2)}{0.8\times 75} + \frac{2}{0.8} = 15\ N/mm^2$$

Number of turns $(n) = \frac{4000 t f_c}{\pi d^2 f_s} = \frac{4000\times 75\times 15}{\pi\times 5^2\times 1000} = 57.29$ turn/m $\simeq 57$

pitch of winding $= \frac{1000}{57} = 17.5\ mm.$

(b) Longitudinal prestressing

critical transient stress at spigot end $= 0.6\times$ hoop stress

$$= 0.6\times 15 = 9\ N/mm^2$$

Maximum permissible tensile stress $= 0.8\sqrt{f_{ck}} = 0.8\times\sqrt{40}$

$$= 5\ N/mm^2$$

Hence the tensile stress of $9-5 = 4\ N/mm^2$ should be counterbalanced by longitudinal prestressing.

cross sectional area of the pipe $= \pi\times 1.075\times 0.075$

$$= 0.253\ m^2.$$

If P is the longitudinal prestressing force required, Then

$$P = \frac{0.253\times 10^6\times 4}{10^3} = \underline{\underline{1013\ kN}}.$$

$\frac{pd}{4t}$

using 7 mm wires stressed to 1000 $N/mm^2$

Force in each wire = 38.5 kN $\left(\frac{\pi}{4}\times 7^2\times 1000\right)$

No. of wires $= \frac{1013}{38.5} = 27$

(c) Check for flexural stresses as per IS 784.

Considering the pipe as a beam of hollow circular section over a span of 6 m.

Three times self wt $= 3\pi \times 1.075 \times 0.075 \times 24 = 18.30$ kN/m

Weight of water $= \frac{\pi \times 1^2 \times 10}{4} = 7.90$ kN/m

Total $w = 26.20$ kN/m.

Max BM $= \frac{26.2 \times 6^2}{8} = 118$ kN-m

$I = \frac{\pi(1.15^4 - 1^4)}{64} = 0.0365\ m^4$

Flexural tensile stress $= \frac{118 \times 10^6 \times 575}{0.0365 \times 10^{12}} = 1.88\ N/mm^2$ (tension)

Longitudinal prestress $= 4\ N/mm^2$

∴ Resultant stress in Concrete $= 4 - 1.88 = 2.12\ N/mm^2$ (Comp)

Since the resultant stress is Comp, pipe is safe against cracking.

(11) Given data

$V = 24500 \times 10^6$ lit

Circumferential

Dia of steel wire = 7 mm ϕ

Initial stress in wire = 1000 $N/mm^2$.

permissible Comp stress in Concrete at transfer $f_c = 13\ N/mm^2$.

$f_{cmin} = 1\ N/mm^2$.

$\eta = 0.75$.

Vertical prestressing

Dia of wire = 8 mm (12 Nos)

Initial stress = 1200 $N/mm^2$.

$f_{ck} = 40\ N/mm^2$. $\mu = 0.5$.

Assume Dia of tank = 50 m. Ht of storage $= h = 12.47 \approx \underline{12.5}$ m

Assume Thickness of the tank wall at the base = 400 mm and reduces to 200 mm at top.

Hydrostatic pressure $W_w = \gamma H = 10 \times 12.5 = 125\ kN/m^2$.

$= 0.125\ N/mm^2$

Max Ring tension (Hoop) $N_d = 10 \times 12.5 \times 25 = 3125$ kN/m.

Self wt of wall $= 12.5 \times 0.3 \times 1 \times 24 = 90$ kN/m.

Frictional force at the base $= 0.5 \times 90 = 45$ kN/m.

Max wall thickness at the base $= \dfrac{3125}{(0.75 \times 13) - (1)} = 360$ mm.

Net thickness available at base $= 400 - 40 = 360$ mm.

Circumferential prestress

$$f_c = \frac{3125}{0.75 \times 360} + \frac{1}{0.75} = 13 \text{ N/mm}^2$$

Spacing of circumferential wire winding

$$S = \frac{2 \times 3125}{0.125} \times \frac{1000 \times 38.5}{13 \times 50 \times 10^3 \times 360} = 8.3 \text{ mm.}$$

No of wires/meter $= 120$

Ring tension at 0.75 m from top $= 10 \times 0.75 \times 25 = 188$ kN/m.

Thickness at top $= 200$ mm.

Net thickness at top $= 200 - 40 = 160$ mm (effective)

$$f_c = \frac{188}{0.75 \times 160} + \frac{1}{0.75} = 2.91 \text{ N/mm}^2.$$

$$S = \frac{2 \times 188}{0.125} \times \frac{1000 \times 38.5}{2.91 \times 50 \times 10^3 \times 160} = 50 \text{mm.}$$

No of wires at top/m $= 20$.

Max residual pressure due to prestress at transfer

$$w_t = \frac{2 \times 1000 \times 38.5}{8.3 \times 50 \times 10^3} = 0.186 \text{ N/mm}^2$$

Max vertical moment due to working pressure is

$$M_t = 35500\left(\frac{0.186}{0.125}\right) = 53000 \text{ N-mm/mm} = 53 \times 10^6 \text{ N-mm/m.}$$

Consider 1m length along circumference

$$Z = \frac{1}{6} \times 1000 \times 400^2 = 26.6 \times 10^6 \text{ mm}^3.$$

Vertical prestress required $f_c = \left[\dfrac{1}{0.75} + \dfrac{53 \times 10^6}{26.6 \times 10^6}\right] = 3.33 \text{ N/mm}^2$

per IS code,

the min vertical prestress required to counteract the winding stress is

$= 0.3 \times 13 = 3.9\ N/mm^2$.

$\therefore$ vertical prestressing force $= \frac{3.9 \times 1000 \times 400}{1000} = 1560$ kN.

spacing of vertical cables $= \frac{1000 \times 720}{1560} = 460$ mm.

Ultimate tensile force in wires at base

$= \frac{120 \times 38.5 \times 1500}{1000} = 6900$ kN.

Load factor against collapse $= \frac{6900}{3125} = 2.2$.

cracking load $= (1000 \times 400) \times \frac{(0.75 \times 13 + 1.7)}{1000} = 4580$ kN.

factor of safety against cracking $= \frac{4580}{3125} = 1.47$.

Nominal reinforcement of 0.2% of the c/s in the circumferential and vertical direction are well distributed on each face.